**Topic 1 Summary: Exponential & Logarithmic Functions**

**The Core Idea: Opposites**

The most important thing to remember is that **exponential functions** and **logarithmic functions** are inverses of each other. They are two sides of the same coin and exist to "undo" one another.

* An **exponential function** tells you how big something gets after a certain amount of growth.
* A **logarithmic function** tells you how much time it took to grow to a certain size.

**1. Exponential Functions (The "Growth" Function)**

This function describes repeated multiplication, which leads to rapid, accelerating growth.

* **General Form:** f(x)=ax**Error! Filename not specified.**
  + a is the **base**: the number you keep multiplying by.
  + x is the **exponent**: how many times you multiply the base.
* **Analogy:** If an investment of ₹1 doubles (a=2) every year, after 3 years (x=3), you would have 23=₹8.
* **Key Graph Feature:** The graph of y=ax always passes through the point **(0, 1)** because anything to the power of 0 is 1.

**2. Logarithmic Functions (The "How Long?" Function)**

This function is the inverse of the exponential. It finds the exponent needed to reach a certain value.

* **General Form:** f(x)=loga​(x)**Error! Filename not specified.**
* **The Question it Answers:** "How many times do I need to multiply a by itself to get x?"
* **Example:** log2​(8)=3 asks, "How many times do we multiply 2 to get 8?" The answer is 3.
* **The Golden Rule (Domain):** The input to a logarithm must be **strictly positive**. You cannot take the log of zero or a negative number.
* **Key Graph Feature:** The graph of y=loga​(x) always passes through the point **(1, 0)** and has a **vertical asymptote** at the y-axis (it gets infinitely close but never touches it).

**3. The "Famous Two" Special Logarithms**

Two bases are so common they have their own names:

* **Common Log (base 10):** Written as log(x). If you see log with no base written, it's always base 10.
* **Natural Log (base *e*):** Written as ln(x). The base is Euler's number, *e* (approx 2.718), which is crucial for describing continuous growth in science and finance.

**4. The Laws of Logarithms (The Shortcuts)**

These are three essential rules that let you simplify and solve equations.

1. **Product Rule:** log(A×B)=log(A)+log(B) *(Multiplying inside a log is the same as adding the logs outside.)*
2. **Quotient Rule:** log(A/B)=log(A)−log(B) *(Dividing inside a log is the same as subtracting the logs outside.)*
3. **Power Rule:** log(An)=n⋅log(A) *(An exponent inside a log can be moved to the front as a multiplier. This is the most important rule for solving equations.)*

**5. The Change of Base Rule (The Practical Tool)**

This formula lets you calculate any logarithm using the log or ln buttons on your calculator.

* **Formula:** loga​(x)=ln(a)ln(x)​ or log(a)log(x)​**Error! Filename not specified.**
* **Example:** To find log5​(89), you simply calculate ln(89) / ln(5).

**6. Main Application: Solving Exponential Equations**

The primary use of logarithms in your course is to solve equations where the variable x is in the exponent. The strategy is always the same: **use the Power Rule to bring the exponent down.**

* **To Solve** 5x=90**:**
  1. Take the natural log (ln) of both sides: ln(5x)=ln(90)**Error! Filename not specified.**
  2. Apply the Power Rule: x⋅ln(5)=ln(90)**Error! Filename not specified.**
  3. Solve for x: x=ln(5)ln(90)​**Error! Filename not specified.**

**7. The Inverse Properties (The Cancellation Rules)**

Because exponentials and logarithms are perfect opposites, they cancel each other out. This gives us two of the most important rules for simplifying expressions.

1. aloga​(x)=x**Error! Filename not specified.**
   * **In Plain English:** This says, "Raise 'a' to the power of the *exact exponent needed to get x*." The result, logically, is just x.
   * **Example:** eln(x)=x**Error! Filename not specified.**
2. loga​(ax)=x**Error! Filename not specified.**
   * **In Plain English:** This asks, "What exponent do you need to raise 'a' to, in order to get the number ax?" The answer is clearly x.
   * **Example:** ln(ex)=x**Error! Filename not specified.**